Indian Res. J. Genet. & Biotech 7(1) : 53 – 60 (2015)

The Effect of Porous Parameter on the unsteady flow of a conducting dusty rivlin-ericksen visco-elastic fluid through the annular space between two circular cylinders and its vorticity

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(Received: November 2014; Revised: December 2015; Accepted: January 2015)

Abstract

In this paper we have investigated the effect of porous parameter on the fluid velocity, dust flow and a transverse magnetic field on the vorticity of flow of an elastic-viscous fluid past the annular space between two circular cylinders. The magnetic lines of force are assumed to be fixed relative to the plate with magnetic field as well as Porous parameter in both the cases. Also we have discussed the applications of these results.

Keywords: Porous, unsteady, conducting, Rivlin-Ericksen, visco-elastic, annular space circular cylinders vorticity.

NOMENCLATURE:

μ	=	viscosity of conducting dusty visco-elastic fluid
±	=	electrical conductivity of the visco-elastic fluid
(r_1, r_2)	=	radii of the inner and outer cylinders respectively.
B_0	=	strength of the magnetic field
(r,¯,z,)	=	cylindrical polar co-ordinates
(u_z, v_z)	=	velocities of the conducting dusty visco-elastic fluid and dust particle
ν	=	kinematic coefficient of viscosity
°	=	density of the fluid
t	=	time
k	=	Stoke's resistance coefficient (for spherical particles of radius a, it is (6 πμa)
N_0	=	number of dust particles as density(taken as constant)
M	=	Hartmann number
r	=	ratio of radii of outer and inner cylinders $\left(\frac{r_2}{r_1}\right)$
f_1	=	mass concentration of the dust particle
$2\left(=\frac{\mu}{k}\right)$	=	Relaxation time parameter of dust particle
K'	=	Porosity of the fluid, $\left(K_1 = \frac{\neg}{K'}\right)$

Introduction

Visco-elastic flows arise in numerous processes in chemical engineering systems. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects. An extensive range of mathematical models has been developed to simulate the diverse hydrodynamic behavior of these

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Published by the Indian Society of Genetics, Biotechnology Research and Development
Biotech Bhawan 5 E Nikhil Estate, DPS Road, Shastripuram, Agra 282007
Online management by www.isgbrd.co.in

non-Newtonian fluid. An eloquent exposition of viscoelastic fluid models has been presented by Joseph [6]. Examples of such models are the Rivlin–Ericksen second order model [7], the Oldroyd model [8], Johnson–Seagalman model [9], the upper convected Maxwell model [10] and the Walters-B model [11]. Both steady and unsteady flows have been investigated at length in a diverse range of geometries using a wide spectrum of analytical and computational methods. Siddappa Khapate [12] studied the second order Rivlin–Ericksen viscoelastic boundary layer flow along a stretching surface. Rochelle and Peddieson [13] used an implicit difference scheme to analyze the steady boundary-layer flow of a nonlinear Maxwell viscoelastic fluid past a parabola and a paraboloid. Rao and Finlayson [14] used an adaptive finite element technique to analyze viscoelastic flow of a Maxwell fluid. Abel et al. [15] investigated the non-Newtonian viscoelastic boundary layer flow of Walter's liquid-B past a stretching sheet, taking account of non-uniform heat source and frictional heating, Abel and Nandeppanavar [16] effects of non-uniform heat source on MHD flow of viscoelastic fluid of Walter's liquid-B. Abel and Mahesha [17] have investigated the effects of thermal conductivity, non-uniform heat source and viscous dissipation in the presence of thermal radiation on the flow and heat transfer in viscoelastic fluid over a stretching sheet, which is subjected to an external magnetic field. Gireesh Kumar and Satyana rayana [18] have examined the mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink.

The study of the dust particle in the motion of the fluid is of great interest for researchers in fluid mechanics. Based on fundamental equations of motion of Saffman [2] studied the laminar flow of dusty gas. Sing and Singh [3] have studied MHD flow of a dusty viscous fluid through an annulus with time dependent pressure gradient. Mitra [1] studied the flow of unsteady conducting dusty fluid through the annular space between the circular cylinders in the presence of transverse magnetic field.

Pillai et al. [19] investigated the effects of work done by deformation in viscoelastic fluid in porous media with uniform heat source, Hayat et al. [20] also investigated the effects of work done by deformation in second grade fluid with partial slip condition, in this no account of heat source has been taken into consideration and Khan et al. [21] also investigated the effect of work done by deformation in Walter's liquid B but with uniform heat source. Sharma et al. [22] have analysed the Rayleigh-Taylor instability of Walter'B elastic-viscous fluid through porous medium. Thermosolutal instability of Walters' (model-B) visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Sharma and Rana [23]. Kesavaiah et al. [24] investigated effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction.

Ramanamurthy et al. [25] have discussed the MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Mustafa et al. [26] obtained the analytical solution of unsteady MHD memory flow with oscillatory suction, variable free stream and heat source. Numerical study of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction was analyzed by Chang et al. [27]. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [28]. Gireesh Kumar et al. [29] analyzed the effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink.

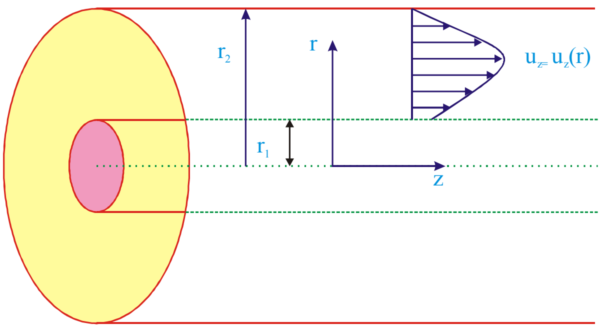
Chowdhury and Islam [30] were studied the MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate. Kafousias and Raptis [31] have discussed the mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction of injection. Raptis et al. [4] have studied the flow of a Walter's liquid B' model in the presence of constant heat flux between the fluid and the plate and taking into account the influence of the memory fluid on the energy equation. MHD free convection flow of an elastoviscous fluid past an infinite vertical plate was analysed by Samria et al. [32].

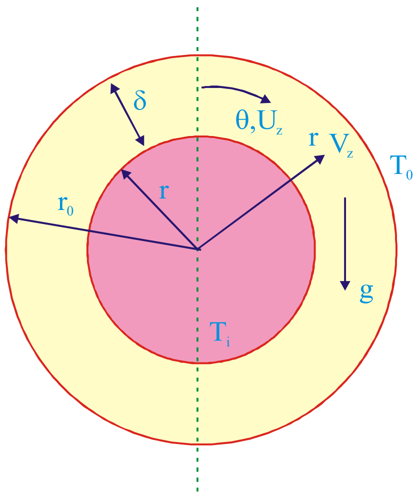
Recently Tyagi and Rajesh [33] considers the flow of a conducting Rivlin-Ericksen elastico-viscous fluid embedded with non conducting identical spherical particle through the annular space between two circular cylinders in presence of transverse magnetic field. Also Rana G.C. [5] studied thermal convection in Rivlin Ericksen rotating fluid permeated with suspended particles in presence of magnetic field and variable gravity in porous medium.

In this paper we have investigated the effect of porosity on the flow of a conducting Rivlin-Ericksen visco elastic fluid through he annular space between two circular cylinders.

FORMILATION OF THE PROBLEM AND ITS SOLUTION

An electrically conducting dusty Rivilin-Ericksen fluid of viscosity 'μ' and electrical conductivity 'σ' is contained between two long co-axial-circular cylinders of radii r_1 and r_2 in the presence of imposed transverse magnetic field of strength B_0. The inner and the outer cylinders are moving with velocities "$\mu_1 e^{\mu_1^2 t}$" and "$\mu_2 e^{\mu_1^2 t}$" respectively. We take a cylindrical polar coordinate system $\left(r, \overline{\ }, z\right)$ by taking z-axis along the axis of the cylinders. The flow of a conducting dustry Rivlin-Ericksen fluid is produced by the motion of the cylinders which moves parallel to z-axis and so there is no displacement of the conducting dusty Rivlin-Ericksen third and dust particles in the direction of 'r' and 'θ' respectively.





Geometry

The equations of motion of this problem are given by

$$\frac{\partial U_z}{\partial t}=\left(\sim+\quad\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 U_z}{\partial r^2}+\frac{1}{r}\frac{\partial U_z}{\partial r}\right)+\frac{KN_0}{\circ}\left(V_z-U_z\right)-\frac{\pm B_0^2}{\circ}U_z-\frac{\neg}{K'}U_z \qquad \text{…(1)}$$

$$\frac{\partial V_z}{\partial t}=\frac{1}{2}\left(U_z-V_z\right) \qquad \text{…(2)}$$

After introducing the non-dimensional quantities

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$$r^*=\frac{r}{r_1},\ t^*=\frac{\neg t}{r_1^2},\ U^*=\frac{U_z r_1}{\neg},\ V^*=\frac{V_z.r_1}{\neg} \qquad \text{…(3)}$$

Applying (3) (The non-dimensional qualities)

Equation (1) and (2) become (after dropping '*')

$$\frac{\partial U}{\partial t} = \left(\sim + A\frac{\partial}{\partial t} \right)\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} \right) + f\left(V - U \right) - \left(M^2 + K_1 \right)U \qquad \ldots(4)$$

$$\frac{\partial V}{\partial t} = \text{a}\left(U - V \right) \qquad \ldots(5)$$

where $f = \frac{f_1}{f_2}\ (say),\quad K_1 = \frac{\neg}{K'}$ Porus parameter

$f_1 = \frac{mN_0}{\circ}$ mass concentration of dust particle

$f_2 = \frac{mV}{Kr^2}$, $\quad 2 = \frac{w}{K}$

$\text{a} = \frac{1}{f_2'}$

$M = B_0 r_1 \sqrt{\pm / \mu}$ (The Hartmann Number)

$A = \frac{\neg}{r_1^2}$

The boundary conditions are:

$$U(r,t) = \mu_1 e^{\mu_1^2 t} \qquad at \qquad r = 1$$

$$V(r,t) = \mu_2 e^{\mu_1^2 t} \qquad at \qquad r = \frac{r_2}{r_1} = \quad (say) > 1 \qquad \ldots(6)$$

Let the solutions of (4) and (5) respectively are

$$U = F(r)e^{\mu_1^2 t} \qquad \ldots(7)$$

$$V = G(r)e^{\mu_1^2 t} \qquad \ldots(8)$$

With the help of (7) and (8) equation (4) and (5) on eliminating v are combined into single ordinary differential equation.

$$\frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr} - S^2 F = 0 \qquad \ldots(9)$$

Where $$S^2 = \left[\frac{\mu_1^2\left(\mu_1^2 + f + \left(M^2 + K_1 \right) \right) + \text{a}^2\left(M^2 + K_1 \right)}{\mu_1^2 + \text{a}} \right]\frac{1}{\left(\sim + A\mu_1^2 \right)} \qquad \ldots(10)$$

Equation (9) is the modified Bessel equation and its solution is

$$F(r) = C_1 I_0(sr) + C_2 K_0(sr) \qquad \ldots(11)$$

Where I_0 and K_0 denote respectively the modified Bessel function of the first and second kind of order zero. Also from (5), (7) and (8),

$$G(r) = \left[\frac{\text{a}}{\mu_1^2 + \text{a}} \right]F(r) \qquad \ldots(12)$$

Thus, from (7) and (8) the velocities of the conducting dusty Rivlin-Ericksen visco-elastic fluid and dust particle are respectively.

$$U = \left[C_1 I_0 (rs) + C_2 K_0 (rs) \right] e^{\mu_1^2 t} \quad \ldots(13)$$

$$And\ V = \left[\frac{a}{\mu_1^2 + a} \right] \left[C_1 I_0 (rs) + C_2 K_0 (rs) \right] e^{\mu_1^2 t} \quad \ldots(14)$$

Applying the boundary condition (6) the values of C_1 and C_2 from (13) and (14) are obtained as

$$C_1 = \frac{\mu_1 K_0 (s\text{¥}) - \mu_2 K_0 (s)}{I_0 (s) K_0 (s\text{¥}) - I_0 (sr) K_0 (s)} \quad \ldots(15)$$

and $$C_2 = \frac{\mu_2 I_0 (s) - \mu_1 I_0 (s\text{¥})}{I_0 (s) K_0 (s\text{¥}) - I_0 (s\text{¥}) K_0 (s)} \quad \ldots(16)$$

Now, let $C_1 = \mu_1 C_1'$

Where $$C_1' = \frac{K_0 (s\text{¥}) - \mu_2 / \mu_1 K_0 (s)}{I_0 (s) K_0 (s\text{¥}) - I_0 (s\text{¥}) K_0 (s)} \quad \ldots(17)$$

And let $C_2' = \mu_2 C_2'$

Where $$C_2' = \frac{\mu_2 / \mu_1 I_0 (s) - I_0 (s\text{¥})}{I_0 (s) K_0 (s\text{¥}) - I_0 (s\text{¥}) K_0 (s)} \quad \ldots(18)$$

From equation (13) we can find the vorticity for the first liquid as:

$$\text{¸}_1 = S \left[C_1' I_1 (rs) + C_2' K_1 (rs) \right] \quad \ldots(19)$$

$\left[K_1 (rs)\ and\ I_1 (rs) \right.$ are modified Bessels functions $\left. \right]$

By giving the different values to S, C_1', C_2', r we get the different values of C_1, $\text{¸}_1 - r$ graph is shown in fig (5) we can find the vorticity for the second liquid as:

$$\text{¸}_2 = \left(\frac{a}{\mu_1^2 + a} \right) S \left[C_1' I_1 (rs) + C_2' K_1 (rs) \right] \quad \ldots(20)$$

By giving the different values to S, C_1', C_2' and r and taking

$$\frac{a}{\mu_1^2 + a} = 0.5$$

A fixed-value, we get the different values of ¸_2–r graph is shown in Fig.-6

RESULTS AND DISCUSSIONS

Fig. 1 – Figure is drawn between fluid velocity U and time t for the different values of porous parameter K_1. This shows that as we increase K_1 initially it is constant but after a certain time interval it increases.

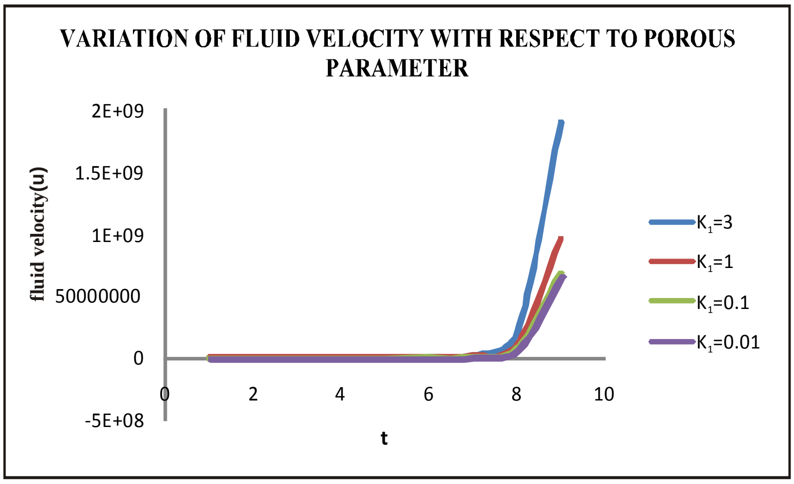


Fig. 2 – Figure is drawn between time t and dust velocity V for the different values of porous parameter K_1 initially it is constant after a certain time interval it increases.

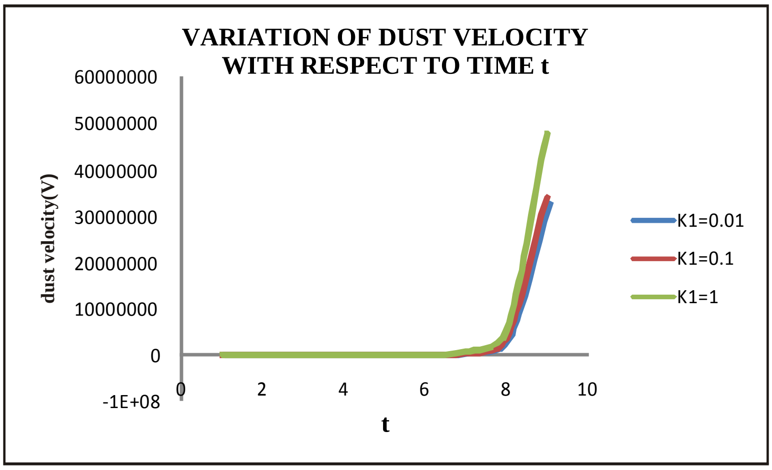


Fig. 3 – Figure is drawn between fluid velocity U and time t for the different values of magnetic parameter M. This shows that initially it is constant but after a certain time interval it increases and after some interval as we increase a small M velocity increases.

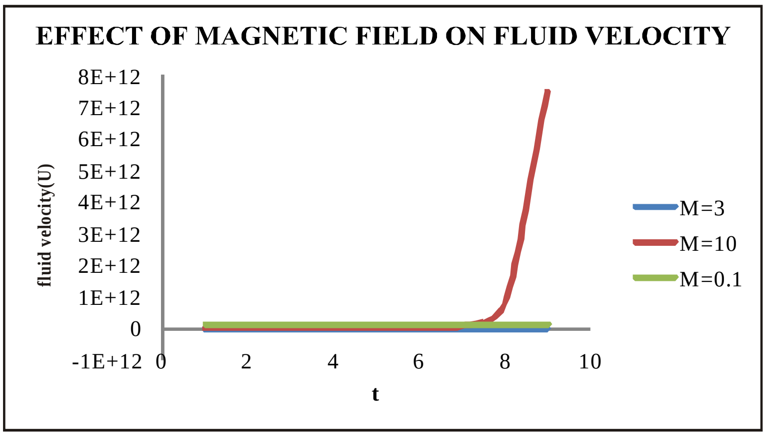


Fig. 4 – Figure is drawn between time t and dust velocity V for the different values of magnetic parameter M. This shows that as we increase M velocity increases after some interval as we increase a small velocity increases.

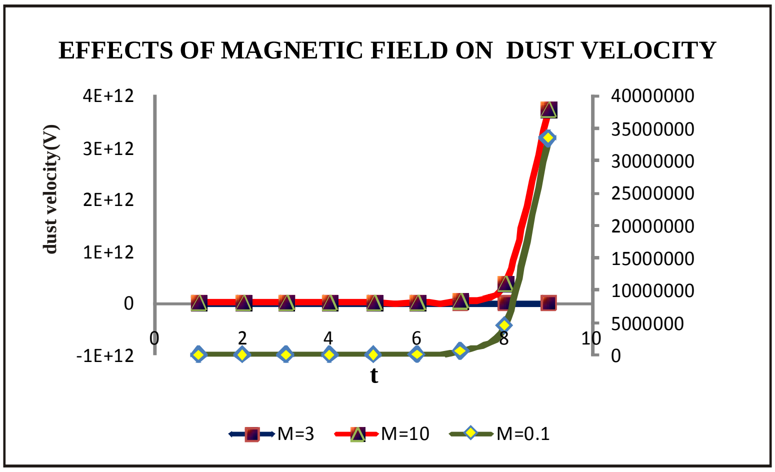


Fig. 5 – Figure is drawn between r and S_1 for the different values of porous parameter K_1. This shows that as we increase K_1, it decreases rapidly.

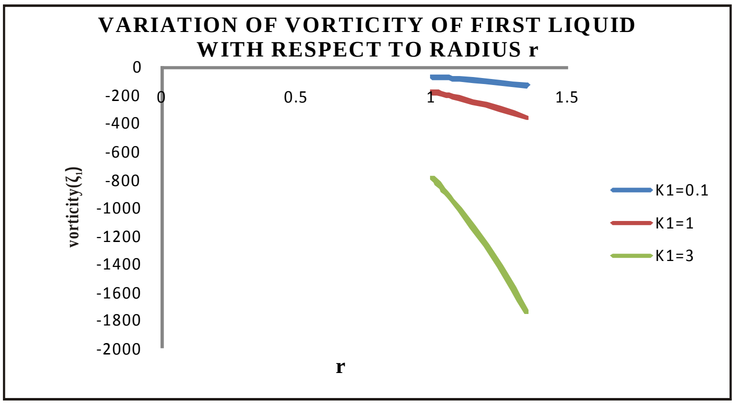
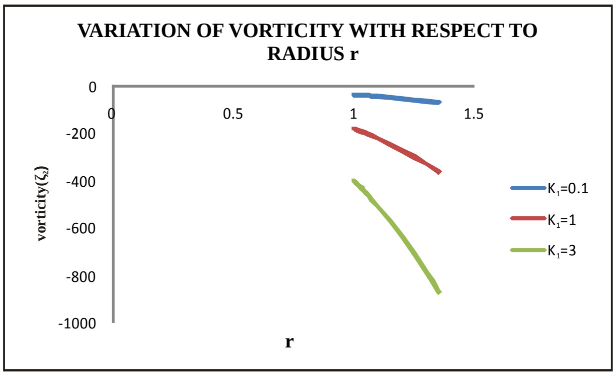


Fig. 6 – Figure is drawn between r and S_2 for the different values of porous parameter. This shows that as we increase K_1. It decreases.



Conclusion

The velocity profiles of fluid and dust velocities are drawn. We observed that

1. Fluid velocity increases with increase in porous parameter as well as with increase in magnetic parater M.
2. Dust velocity increases with increase in porous parameter as well as increase in magnetic parameter M.
3. The vorticity for the two fluids in the two cylindrical parts decrease with increase the porous parameter K_1.

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